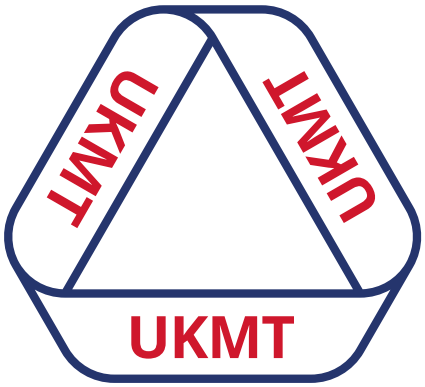
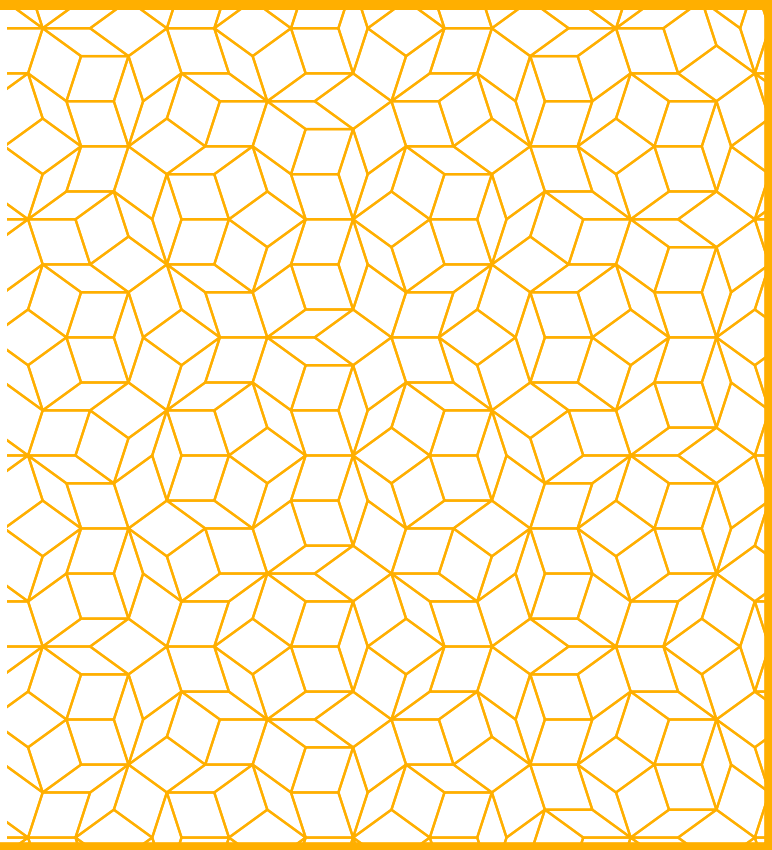
Junior Mathematical Challenge

Past Papers and Solutions 2010-2014



United Kingdom Mathematics Trust



UK JUNIOR MATHEMATICAL CHALLENGE

THURSDAY 29th APRIL 2010

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

The Actuarial Profession
making financial sense of the future

RULES AND GUIDELINES (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 8 or below.
   Candidates in Scotland must be in S2 or below.
   Candidates in Northern Ireland must be in School Year 9 or below.
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9. The questions on this paper challenge you to **think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.
   The UK JMC is about solving interesting problems, not about lucky guessing.

The UKMT is a registered charity

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1. What is 2010 + (+2010) + (−2010) − (+2010) − (−2010) ?

A 0 B 2010 C 4020 D 6030 E 8040

2. Each letter in the abbreviation shown is rotated through 90° clockwise. Which of the following could be the result?

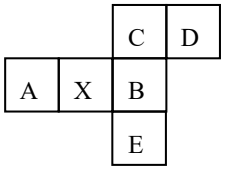
U K M T

A B C D E

3. Which of the following could have a length of 2010 mm?

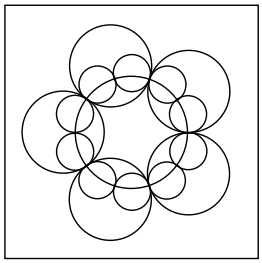
A a table B an oil tanker C a teaspoon D a school hall E a hen's egg

4. If the net shown is folded to make a cube, which letter is opposite X ?



A B C D E

5. The diagram shows a pattern of 16 circles inside a square.
The central circle passes through the points where the other circles touch.
The circles divide the square into regions. How many regions are there?



A 17 B 26 C 30 D 32 E 38

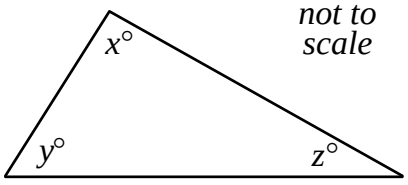
6. Which of the following has the largest value?

A $6 \div \frac{1}{2}$ B $5 \div \frac{1}{3}$ C $4 \div \frac{1}{4}$ D $3 \div \frac{1}{5}$ E $2 \div \frac{1}{6}$

7. Mr Owens wants to keep the students quiet during a Mathematics lesson. He asks them to multiply all the numbers from 1 to 99 together and then tell him the last-but-one digit of the result. What is the correct answer?

A 0 B 1 C 2 D 8 E 9

8. In a triangle with angles $x°$, $y°$, $z°$ the mean of y and z is x.
What is the value of x?



A 90 B 80 C 70 D 60 E 50

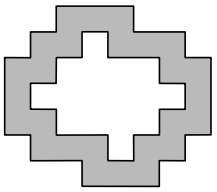
9. Which of the following is the longest period of time?

A 3002 hours B 125 days C $17\frac{1}{2}$ weeks D 4 months E $\frac{1}{3}$ of a year

10. At the Marldon Apple-Pie-Fayre bake-off, prize money is awarded for 1st, 2nd and 3rd places in the ratio 3 : 2 : 1. Last year Mrs Keat and Mr Jewell shared third prize equally. What fraction of the total prize money did Mrs Keat receive?

A $\frac{1}{4}$ B $\frac{1}{5}$ C $\frac{1}{6}$ D $\frac{1}{10}$ E $\frac{1}{12}$

11. In the diagram shown, all the angles are right angles and all the sides are of length 1 unit, 2 units or 3 units.
What, in square units, is the area of the shaded region?

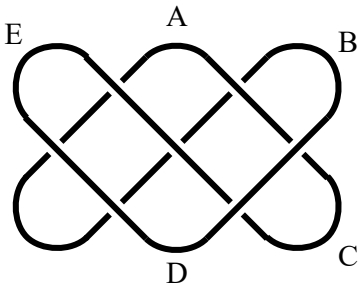


A 22 B 24 C 26 D 28 E 30

12. Sir Lance has a lot of tables and chairs in his house. Each rectangular table seats eight people and each round table seats five people. What is the smallest number of tables he will need to use to seat 35 guests and himself, without any of the seating around these tables remaining unoccupied?

A 4 B 5 C 6 D 7 E 8

13. The diagram shows a Lusona, a sand picture of the Tshokwe people from the West Central Bantu area of Africa. To draw a Lusona the artist uses a stick to draw a single line in the sand, starting and ending in the same place without lifting the stick in between. At which point could this Lusona have started?

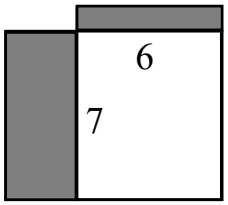


A B C D E

14. The Severn Bridge has carried just over 300 million vehicles since it was opened in 1966. On average, roughly how many vehicles is this per day?

A 600 B 2 000 C 6 000 D 20 000 E 60 000

15. A 6 by 8 and a 7 by 9 rectangle overlap with one corner coinciding as shown.
What is the area (in square units) of the region *outside* the overlap?

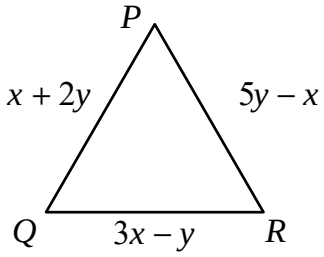


A 6 B 21 C 27 D 42 E 69

16. One of the examination papers for Amy's Advanced Arithmetic Award was worth 18% of the final total. The maximum possible mark on this paper was 108 marks. How many marks were available overall?

A 420 B 480 C 540 D 560 E 600

17. The lengths, in cm, of the sides of the equilateral triangle PQR are as shown.
Which of the following could *not* be the values of x and y?

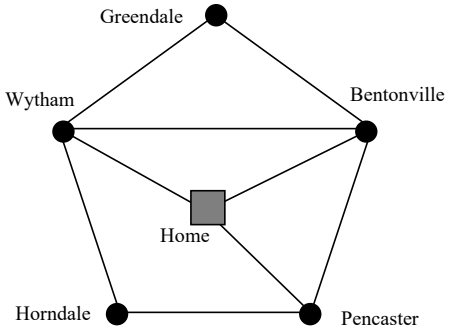


A (18, 12) B (15, 10) C (12, 8) D (10, 6) E (3, 2)

18. Sam's 101st birthday is tomorrow. So Sam's age in years changes from a square number (100) to a prime number (101). How many times has this happened before in Sam's lifetime?

A 1 B 2 C 3 D 4 E 5

19. Pat needs to travel down every one of the roads shown at least once, starting and finishing at home. What is the smallest number of the five villages that Pat will have to visit more than once?

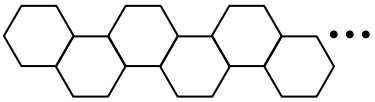


A 1 B 2 C 3 D 4 E 5

20. Nicky has to choose 7 different positive whole numbers whose mean is 7. What is the largest possible such number she could choose?

A 7 B 28 C 34 D 43 E 49

21. A shape consisting of a number of regular hexagons is made by continuing to the right the pattern shown in the diagram, with each extra hexagon sharing one side with the preceding one. Each hexagon has a side length of 1 cm. How many hexagons are required for the perimeter of the whole shape to have length 2010 cm?

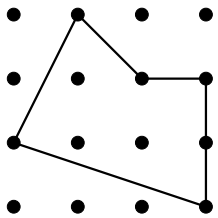


A 335 B 402 C 502 D 670 E 1005

22. Kiran writes down six different prime numbers, p, q, r, s, t, u, all less than 20, such that $p + q = r + s = t + u$. What is the value of $p + q$?

A 16 B 18 C 20 D 22 E 24

23. A single polygon is made by joining dots in the 4×4 grid with straight lines, which meet only at dots at their end points. No dot is at more than one corner. The diagram shows a five-sided polygon formed in this way. What is the greatest possible number of sides of a polygon formed by joining the dots using these same rules?



A 12 B 13 C 14 D 15 E 16

24. The year 2010 belongs to a special sequence of twenty-five consecutive years: each number from 1988 to 2012 contains a repeated digit.

Each of the following belongs to a sequence of consecutive years, where each number in the sequence contains at least one repeated digit.

Which of them belongs to the next such sequence of at least twenty years?

A 2099 B 2120 C 2199 D 2989 E 3299

25. What is the value of $P + Q + R$ in the multiplication on the right?

A 13 B 12 C 11 D 10 E 9

$$\begin{array}{cccccc} & & P & Q & P & Q \\ & \times & & R & R & R \\ \hline 6 & 3 & 9 & 0 & 2 & 7 \end{array}$$

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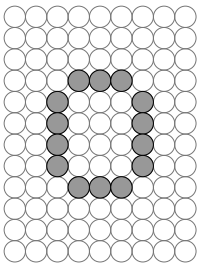
1. What is the value of $2 \times 0 \times 1 + 1$?

A 0 B 1 C 2 D 3 E 4

2. How many of the integers 123, 234, 345, 456, 567 are multiples of 3?

A 1 B 2 C 3 D 4 E 5

3. A train display shows letters by lighting cells in a grid, such as the letter 'o' shown. A letter is made **bold** by also lighting any unlit cell immediately to the right of one in the normal letter. How many cells are lit in a **bold** 'o'?



A 22 B 24 C 26 D 28 E 30

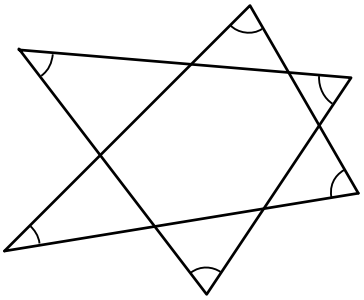
4. The world's largest coin, made by the Royal Mint of Canada, was auctioned in June 2010. The coin has mass 100 kg, whereas a standard British £1 coin has mass 10 g. What sum of money in £1 coins has the same mass as the record-breaking coin?

A £100 B £1000 C £10 000 D £100 000 E £1 000 000

5. All old Mother Hubbard had in her cupboard was a Giant Bear chocolate bar. She gave each of her children one-twelfth of the chocolate bar. One third of the bar was left. How many children did she have?

A 6 B 8 C 12 D 15 E 18

6. What is the sum of the marked angles in the diagram?

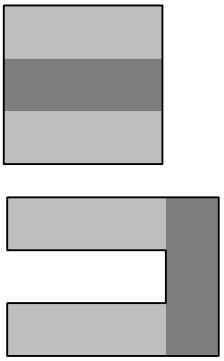


A 90° B 180° C 240° D 300° E 360°

7. Peter Piper picked a peck of pickled peppers. 1 peck = $\frac{1}{4}$ bushel and 1 bushel = $\frac{1}{9}$ barrel. How many **more** pecks must Peter Piper pick to fill a barrel?

A 12 B 13 C 34 D 35 E 36

8. A square is divided into three congruent rectangles. The middle rectangle is removed and replaced on the side of the original square to form an octagon as shown. What is the ratio of the length of the perimeter of the square to the length of the perimeter of the octagon?

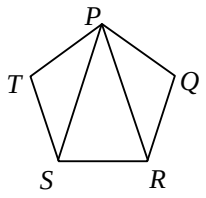


A 3:5 B 2:3 C 5:8 D 1:2 E 1:1

9. What is the smallest possible difference between two different nine-digit integers, each of which includes all of the digits 1 to 9?

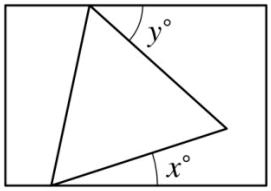
A 9 B 18 C 27 D 36 E 45

10. You want to draw the shape on the right without taking your pen off the paper and without going over any line more than once. Where should you start?



A only at T or Q B only at P C only at S or R
D at any point E the task is impossible

11. The diagram shows an equilateral triangle inside a rectangle.
What is the value of $x + y$?



A 30 B 45 C 60 D 75 E 90

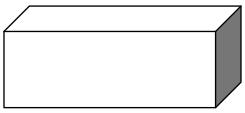
12. If ▲ + ▲ = ■ and ■ + ▲ = ● and ♦ = ● + ■ + ▲, how many ▲s are equal to ♦ ?

A 2 B 3 C 4 D 5 E 6

13. What is the mean of $\frac{2}{3}$ and $\frac{4}{9}$?

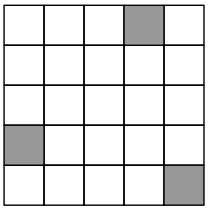
A $\frac{1}{2}$ B $\frac{2}{9}$ C $\frac{7}{9}$ D $\frac{3}{4}$ E $\frac{5}{9}$

14. The diagram shows a cuboid in which the area of the shaded face is one-quarter of the area of each of the two visible unshaded faces.
The total surface area of the cuboid is 72 cm^2. What, in cm^2, is the area of one of the visible unshaded faces of the cuboid?



A 16 B 28.8 C 32 D 36 E 48

15. What is the smallest number of *additional* squares which must be shaded so that this figure has at least one line of symmetry *and* rotational symmetry of order 2 ?



A 3 B 5 C 7 D 9 E more than 9

16. The pupils in Year 8 are holding a mock election. A candidate receiving more votes than any other wins. The four candidates receive 83 votes between them. What is the smallest number of votes the winner could receive?

A 21 B 22 C 23 D 41 E 42

17. Last year's match at Wimbledon between John Isner and Nicolas Mahut, which lasted 11 hours and 5 minutes, set a record for the longest match in tennis history. The fifth set of the match lasted 8 hours and 11 minutes.
Approximately what fraction of the whole match was taken up by the fifth set?

A $\frac{1}{5}$ B $\frac{2}{5}$ C $\frac{3}{5}$ D $\frac{3}{4}$ E $\frac{9}{10}$

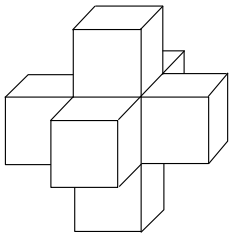
18. Peri the winkle leaves on Monday to go and visit Granny, 90m away. Except for rest days, Peri travels 1m each day (24-hour period) at a constant rate and without pause. However, Peri stops for a 24-hour rest every tenth day, that is, after every nine days' travelling. On which day of the week does Peri arrive at Granny's?

A Sunday B Monday C Tuesday D Wednesday E Thursday

19. A list is made of every digit that is the units digit of at least one prime number. How many of the following numbers appear in the list?

A 1 B 2 C 3 D 4 E 5

20. One cube has each of its faces covered by one face of an identical cube, making a solid as shown. The volume of the solid is 875 cm^3. What, in cm^2, is the surface area of the solid?



A 750 B 800 C 875 D 900 E 1050

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21. Gill leaves Lille by train at 09:00. The train travels the first 27 km at 96 km/h. It then stops at Lens for 3 minutes before travelling the final 29 km to Lillers at 96 km/h. At what time does Gill arrive at Lillers?

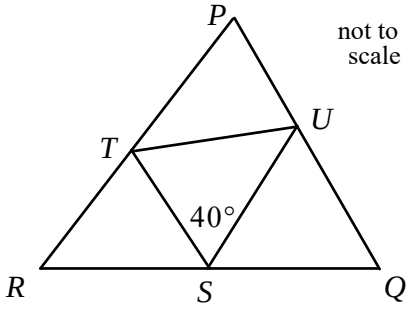
A 09:35 B 09:38 C 09:40 D 09:41 E 09:43

22. Last week Evariste and Sophie both bought some stamps for their collections. Each stamp Evariste bought cost him £1.10, whilst Sophie paid 70p for each of her stamps. Between them they spent exactly £10. How many stamps did they buy in total?

A 9 B 10 C 11 D 12 E 13

23. The points S, T, U lie on the sides of the triangle PQR, as shown, so that $QS = QU$ and $RS = RT$.
$\angle TSU = 40°$. What is the size of $\angle TPU$?

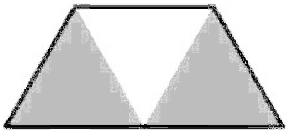
A 60° B 70° C 80° D 90° E 100°



24. Two adults and two children wish to cross a river. They make a raft but it will carry only the weight of one adult or two children. What is the minimum number of times the raft must cross the river to get all four people to the other side? (N.B. The raft may not cross the river without at least one person on board.)

A 3 B 5 C 7 D 9 E 11

25. The diagram shows a trapezium made from three equilateral triangles.
Three copies of the trapezium are placed together, without gaps or overlaps and so that only complete edges coincide, to form a polygon with N sides.
How many different values of N are possible?



A 4 B 5 C 6 D 7 E 8

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1. What is the smallest four-digit positive integer which has four different digits?

   A 1032 B 2012 C 1021 D 1234 E 1023

2. What is half of 1.01?

   A 5.5 B 0.55 C 0.505 D 0.5005 E 0.055

3. Which of the following has exactly one factor other than 1 and itself?

   A 6 B 8 C 13 D 19 E 25

4. Beatrix looks at the word JUNIOR in a mirror.
   How many of the reflected letters never look the same as the original, no matter how Beatrix holds the mirror?

   A 1 B 2 C 3 D 4 E 5

5. One of the mascots for the 2012 Olympic Games is called 'Wenlock' because the town of Wenlock in Shropshire first held the Wenlock Olympian Games in 1850. How many years ago was that?

   A 62 B 152 C 158 D 162 E 172

6. The diagrams on the right show three different views of the same cube. Which letter is on the face opposite U?

   A I B P C K D M E O

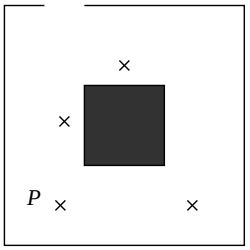
7. A small ink cartridge has enough ink to print 600 pages. Three small cartridges can print as many pages as two medium cartridges. Three medium cartridges can print as many pages as two large cartridges. How many pages can be printed using a large cartridge?

   A 1200 B 1350 C 1800 D 2400 E 5400

8. Tommy Thomas's tankard holds 480ml when it is one quarter empty. How much does it hold when it is one quarter full?

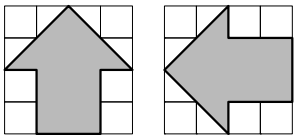
   A 120 ml B 160 ml C 240 ml D 960 ml E 1440 ml

9. The diagram on the right shows the positions of four people (each marked ×) in an Art Gallery. In the middle of the room is a stone column. Ali can see none of the other three people. Bea can see only Caz. Caz can see Bea and Dan. Dan can see only Caz.
   Who is at position *P*?



   A Ali B Bea C Caz
   D Dan E More information needed

10. The diagram shows two arrows drawn on separate 4 cm × 4 cm grids. One arrow points North and the other points West.
    When the two arrows are drawn on the same 4 cm × 4 cm grid (still pointing North and West) they overlap. What is the area of overlap?



    A 4 cm² B 4½ cm² C 5 cm² D 5½ cm² E 6 cm²

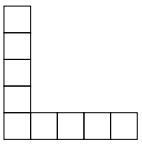
11. In the following expression, each ☐ is to be replaced with either + or – in such a way that the result of the calculation is 100.

123 ☐ 45 ☐ 67 ☐ 89

The number of + signs used is p and the number of – signs used is m. What is the value of $p - m$?

A −3 B −1 C 0 D 1 E 3

12. Laura wishes to cut this shape, which is made up of nine small squares, into pieces that she can then rearrange to make a 3 × 3 square.



What is the smallest number of pieces that she needs to cut the shape into so that she can do this?

A 2 B 3 C 4 D 5 E 6

13. In the multiplication grid on the right, the input factors (in the first row and the first column) are all missing and only some of the products within the table have been given.

What is the value of A + B + C + D + E?

A 132 B 145 C 161 D 178 E 193

×					
	A	10		20	
	15	B	40		
	18		C	60	
		20		D	24
			56		E

14. A pattern that repeats every six symbols starts as shown below:

♥ ♣ ♥ ♦ ♥ ♠ ♥ ♣ ♥ ♦ ♥ ♠ …

Which are the 100th and 101st symbols, in that order, in the pattern?

A ♦ ♥ B ♥ ♦ C ♥ ♣ D ♠ ♥ E ♣ ♥

15. Talulah plants 60 tulip bulbs. When they flower, she notes that half are yellow; one third of those which are not yellow are red; and one quarter of those which are neither yellow nor red are pink. The remainder are white. What fraction of the tulips are white?

A $\frac{1}{24}$ B $\frac{1}{12}$ C $\frac{1}{6}$ D $\frac{1}{5}$ E $\frac{1}{4}$

16. Beth, Carolyn and George love reading their favourite bedtime story together. They take it in turns to read a page, always in the order Beth, then Carolyn, then George. All twenty pages of the story are read on each occasion. One evening, Beth is staying at Grandma's house but Carolyn and George still read the same bedtime story and take it in turns to read a page with Carolyn reading the first page.

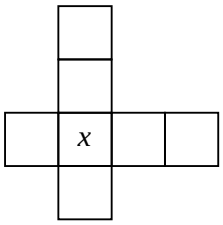
In total, how many pages are read by the person who usually reads that page?

A 1 B 2 C 4 D 6 E 7

17. There are six more girls than boys in Miss Spelling's class of 24 pupils. What is the ratio of girls to boys in this class?

A 5:3 B 4:1 C 3:1 D 1:4 E 3:5

18. The numbers 2, 3, 4, 5, 6, 7, 8 are to be placed, one per square, in the diagram shown such that the four numbers in the horizontal row add up to 21 and the four numbers in the vertical column also add up to 21.
Which number should replace x ?



A 2 B 3 C 5 D 7 E 8

19. In rectangle $PQRS$, the ratio of $\angle PSQ$ to $\angle PQS$ is 1:5. What is the size of $\angle QSR$?

A 15° B 18° C 45° D 72° E 75°

20. Aroon says his age is 50 years, 50 months, 50 weeks and 50 days old. What age will he be on his next birthday?

A 56 B 55 C 54 D 53 E 51

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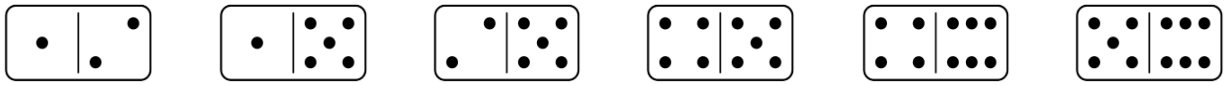
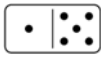
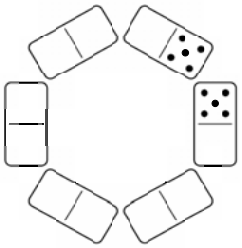
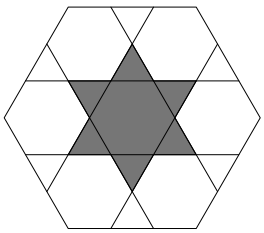
21. 

Dominic wants to place the six dominoes above in a hexagonal ring so that, for every pair of adjacent dominoes, the numbers of pips match. The ring on the right indicates how one adjacent pair match.
In a completed ring, how many of the other five dominoes can he definitely *not* place adjacent to  ?



A 1 B 2 C 3 D 4 E 5

22. The diagram shows a design formed by drawing six lines in a regular hexagon. The lines divide each edge of the hexagon into three equal parts.
What fraction of the hexagon is shaded?



A $\frac{1}{5}$ B $\frac{2}{9}$ C $\frac{1}{4}$ D $\frac{3}{10}$ E $\frac{5}{16}$

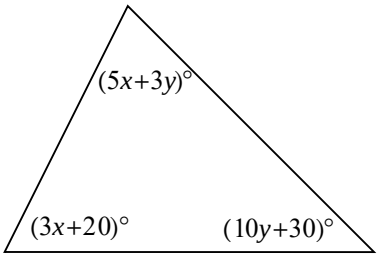
23. Peter wrote a list of all the numbers that could be produced by changing one digit of the number 200. How many of the numbers on Peter's list are prime?

A 0 B 1 C 2 D 3 E 4

24. After playing 500 games, my success rate at Spider Solitaire is 49%. Assuming I win every game from now on, how many extra games do I need to play in order that my success rate increases to 50% ?

A 1 B 2 C 5 D 10 E 50

25. The interior angles of a triangle are $(5x + 3y)°$, $(3x + 20)°$ and $(10y + 30)°$ where x, y are positive integers.
What is the value of $x + y$?



A 15 B 14 C 13 D 12 E 11

UK JUNIOR MATHEMATICAL CHALLENGE

THURSDAY 25th APRIL 2013

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

Institute and Faculty of Actuaries

RULES AND GUIDELINES (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
   No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 8 or below.
   Candidates in Scotland must be in S2 or below.
   Candidates in Northern Ireland must be in School Year 9 or below.
5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
6. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
7. Five marks are awarded for each correct answer to Questions 1-15.
   Six marks are awarded for each correct answer to Questions 16-25.
   Each incorrect answer to Questions 16-20 loses 1 mark.
   Each incorrect answer to Questions 21-25 loses 2 marks.
8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the sheet except to mark your chosen options**. The machine 'sees' all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.
9. The questions on this paper challenge you to **think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers.
   The UK JMC is about solving interesting problems, not about lucky guessing.

The UKMT is a registered charity

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1. Which of the following has the largest value?

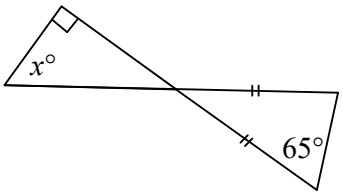
A $1 - 0.1$ B $1 - 0.01$ C $1 - 0.001$ D $1 - 0.0001$ E $1 - 0.00001$

2. Heidi is 2.1 m tall, while Lola is only 1.4 m tall. What is their average height?

A 1.525 m B 1.6 m C 1.7 m D 1.725 m E 1.75 m

3. What is the value of x?

A 25 B 35 C 40 D 65 E 155



4. Gill went for a five-hour walk. Her average speed was between 3 km/h and 4 km/h. Which of the following could be the distance she walked?

A 12 km B 14 km C 19 km D 24 km E 35 km

5. The diagram shows a weaver's design for a *rihlèlò*, a winnowing tray from Mozambique.
How many lines of symmetry does the design have?

A 0 B 1 C 2 D 4 E 8

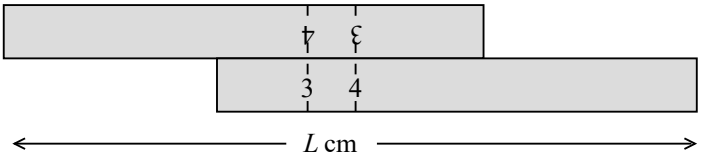
6. What is the value of $((1 - 1) - 1) - (1 - (1 - 1))$?

A -2 B -1 C 0 D 1 E 2

7. After tennis training, Andy collects twice as many balls as Roger and five more than Maria. They collect 35 balls in total. How many balls does Andy collect?

A 20 B 19 C 18 D 16 E 8

8. Two identical rulers are placed together, as shown (not to scale).
Each ruler is exactly 10 cm long and is marked in centimetres from 0 to 10. The 3 cm mark on each ruler is aligned with the 4 cm mark on the other.



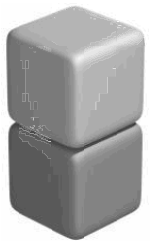
The overall length is L cm. What is the value of L?

A 13 B 14 C 15 D 16 E 17

9. Peter has three times as many sisters as brothers. His sister Louise has twice as many sisters as brothers. How many children are there in the family?

A 15 B 13 C 11 D 9 E 5

10. On standard dice the total number of pips on each pair of opposite faces is 7.
Two standard dice are placed in a stack, as shown, so that the total number of pips on the two touching faces is 5.
What is the total number of pips on the top and bottom faces of the stack?

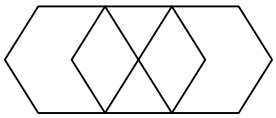


A 5 B 6 C 7 D 8 E 9

11. Usain runs twice as fast as his mum. His mum runs five times as fast as his pet tortoise, Turbo. They all set off together for a run down the same straight path. When Usain has run 100 m, how far apart are his mum and Turbo the tortoise?

A 5 m B 10 m C 40 m D 50 m E 55 m

12. How many hexagons are there in the diagram?



A 4 B 6 C 8 D 10 E 12

13. When painting the lounge, I used half of a 3 litre can to complete the first coat of paint. I then used two thirds of what was left to complete the second coat. How much paint was left after both coats were complete?

A 150 ml B 200 ml C 250 ml D 500 ml E 600 ml

14. Each side of an isosceles triangle is a whole number of centimetres. Its perimeter has length 20 cm. How many possibilities are there for the lengths of its sides?

A 3 B 4 C 5 D 6 E 7

15. The Grand Old Duke of York had 10 000 men. He lost 10% of them on the way to the top of the hill, and he lost 15% of the rest as he marched them back down the hill. What percentage of the 10 000 men were still there when they reached the bottom of the hill?

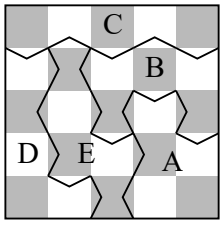
A $76\frac{1}{2}\%$ B 75% C $73\frac{1}{2}\%$ D $66\frac{2}{3}\%$ E 25%

---

16. Ulysses, Kim, Mei and Tanika have their 12th, 14th, 15th and 15th birthdays today. In what year will their ages first total 100?

A 2023 B 2024 C 2025 D 2057 E 2113

17. A 5 cm × 5 cm square is cut into five pieces, as shown.
Each cut is a sequence of identical copies of the same shape but pointing up, down, left or right.
Which piece has the longest perimeter?



A B C D E

18. Weighing the baby at the clinic was a problem. The baby would not keep still and caused the scales to wobble. So I held the baby and stood on the scales while the nurse read off 78 kg. Then the nurse held the baby while I read off 69 kg. Finally I held the nurse while the baby read off 137 kg. What was the combined weight of all three ?

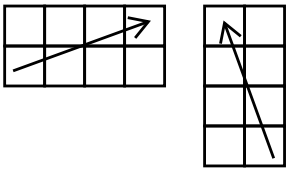
A 142 kg B 147 kg C 206 kg D 215 kg E 284 kg

(*This problem appeared in the first Schools' Mathematical Challenge in 1988 − 25 years ago.*)

19. A swimming club has three categories of members: junior, senior, veteran. The ratio of junior to senior members is 3 : 2 and the ratio of senior members to veterans is 5 : 2.
Which of the following could be the total number of members in the swimming club?

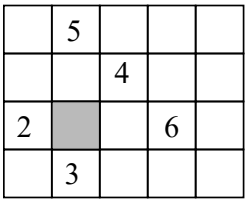
A 30 B 35 C 48 D 58 E 60

20. A 'long knight' moves on a square grid. A single move, as shown, consists of moving three squares in one direction (horizontally or vertically) and then one square at right angles to the first direction. What is the smallest number of moves a long knight requires to go from one corner of an 8 × 8 square board to the diagonally opposite corner?



A 4 B 5 C 6 D 7 E 8

21. The 5 × 4 grid is divided into blocks. Each block is a square or a rectangle and contains the number of cells indicated by the integer within it.
Which integer will be in the same block as the shaded cell?



A 2 B 3 C 4 D 5 E 6

22 . Two numbers in the 4 × 4 grid can be swapped to create a Magic Square (in which all rows, all columns and both main diagonals add to the same total).
What is the sum of these two numbers?

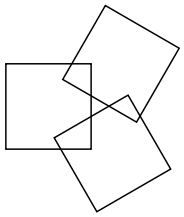
9	6	3	16
4	13	10	5
14	1	8	11
7	12	15	2

A 12 B 15 C 22 D 26 E 28

23. In our school netball league a team gains a certain whole number of points if it wins a game, a lower whole number of points if it draws a game and no points if it loses a game. After 10 games my team has won 7 games, drawn 3 and gained 44 points. My sister's team has won 5 games, drawn 2 and lost 3. How many points has her team gained?

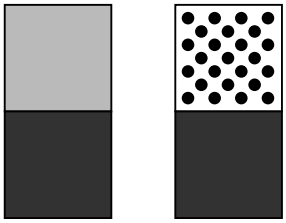
A 28 B 29 C 30 D 31 E 32

24. Three congruent squares overlap as shown. The areas of the three overlapping sections are 2 cm^2, 5 cm^2 and 8 cm^2 respectively. The total area of the non-overlapping parts of the squares is 117 cm^2.
What is the side-length of each square?



A 6 cm B 7 cm C 8 cm D 9 cm E 10 cm

25. For Beatrix's latest art installation, she has fixed a 2 × 2 square sheet of steel to a wall. She has two 1 × 2 magnetic tiles, both of which she attaches to the steel sheet, in any orientation, so that none of the sheet is visible and the line separating the two tiles cannot be seen. As shown alongside, one tile has one black cell and one grey cell; the other tile has one black cell and one spotted cell.
How many different looking 2 × 2 installations can Beatrix obtain?



A 4 B 8 C 12 D 14 E 24

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1. What is $(999 - 99 + 9) \div 9$?

A 91 B 99 C 100 D 101 E 109

2. How many minutes are there in $\frac{1}{12}$ of a day?

A 240 B 120 C 60 D 30 E 15

3. In my row in the theatre the seats are numbered consecutively from T1 to T50. I am sitting in seat T17 and you are sitting in seat T39. How many seats are there between us?

A 23 B 22 C 21 D 20 E 19

4. The number 987 654 321 is multiplied by 9. How many times does the digit 8 occur in the result?

A 1 B 2 C 3 D 4 E 9

5. What is the difference between the smallest 4-digit number and the largest 3-digit number?

A 1 B 10 C 100 D 1000 E 9899

6. The diagram shows a square divided into strips of equal width. Three strips are black and two are grey. What fraction of the perimeter of the square is grey?

A $\frac{1}{5}$ B $\frac{1}{4}$ C $\frac{4}{25}$ D $\frac{1}{3}$ E $\frac{2}{5}$

7. What is $2014 - 4102$?

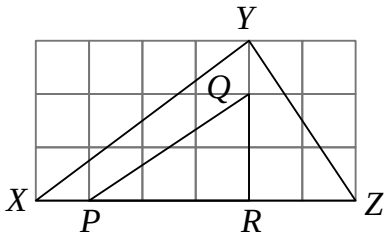
A -2012 B -2088 C -2092 D -2098 E -2112

8. How many prime numbers are there in the list

1, 12, 123, 1234, 12 345, 123 456 ?

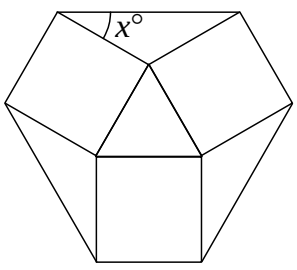
A 0 B 1 C 2 D 3 E 4

9. Triangles XYZ and PQR are drawn on a square grid. What fraction of the area of triangle XYZ is the area of triangle PQR?



A $\frac{1}{4}$ B $\frac{7}{18}$ C $\frac{1}{2}$ D $\frac{5}{18}$ E $\frac{1}{3}$

10. An equilateral triangle is surrounded by three squares, as shown. What is the value of x?



A 15 B 18 C 24 D 30 E 36

11. The first two terms of a sequence are 1 and 2. Each of the following terms in the sequence is the sum of all the terms which come before it in the sequence.
Which of these is *not* a term in the sequence?

A 6 B 24 C 48 D 72 E 96

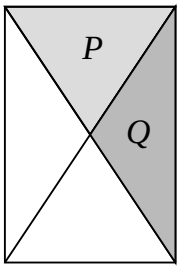
12. In this subtraction, P, Q, R, S and T represent single digits.

$$\begin{array}{rrrrrr} & 7 & Q & 2 & S & T \\ - & P & 3 & R & 9 & 6 \\ \hline & 2 & 2 & 2 & 2 & 2 \end{array}$$

What is the value of $P + Q + R + S + T$?

A 30 B 29 C 28 D 27 E 26

13. A rectangle is split into triangles by drawing in its diagonals.
What is the ratio of the area of triangle P to the area of triangle Q?



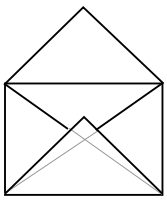
A 1 : 1 B 1 : 2 C 2 : 1 D 2 : 3

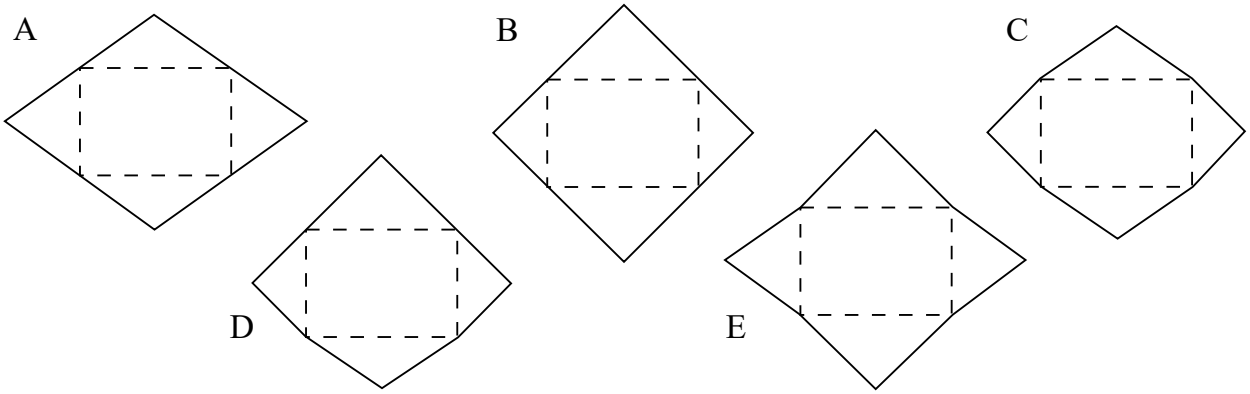
E the ratio depends on the lengths of the sides of the rectangle

14. Which of these is equal to one million millimetres?

A 1 metre B 10 metres C 100 metres D 1 kilometre E 10 kilometres

15. The diagram shows a rectangular envelope made by folding (and gluing) a single piece of paper.
What could the original unfolded piece of paper look like?
(The dashed lines are the fold lines.)

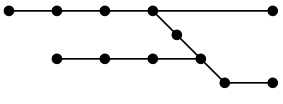




16. Only one of the following statements is true. Which one?

A 'B is true' B 'E is false' C 'Statements A to E are true'
D 'Statements A to E are false' E 'A is false'

17. The diagram is a 'map' of Jo's local rail network, where the dots represent stations and the lines are routes. Jo wants to visit all the stations, travelling only by train, starting at any station and ending at any station, with no restrictions on which routes are taken.
What is the smallest number of stations that Jo must go to more than once?



A 1 B 2 C 3 D 4 E 5

18. Which of these statements is true?

A $15\,614 = 1 + 5^6 - 1 \times 4$ B $15\,615 = 1 + 5^6 - 1 \times 5$ C $15\,616 = 1 + 5^6 - 1 \times 6$
D $15\,617 = 1 + 5^6 - 1 \times 7$ E $15\,618 = 1 + 5^6 - 1 \times 8$

19. Jack and Jill played a game for two people. In each game, the winner was awarded 2 points and the loser 1 point. No games were drawn. Jack won exactly 4 games and Jill had a final score of 10 points. How many games did they play?

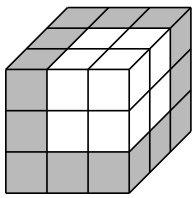
A 5 B 6 C 7 D 8 E impossible to determine

20. Box P has p chocolates and box Q has q chocolates, where p and q are both odd and $p > q$. What is the smallest number of chocolates which would have to be moved from box P to box Q so that box Q has more chocolates than box P?

A $\frac{q-p+2}{2}$ B $\frac{p-q+2}{2}$ C $\frac{q+p-2}{2}$ D $\frac{p-q-2}{2}$ E $\frac{q+p+2}{2}$

---

21. Pablo's teacher has given him 27 identical white cubes. She asks him to paint some of the faces of these cubes grey and then stack the cubes so that they appear as shown. What is the largest possible number of the individual white cubes which Pablo can leave with no faces painted grey?

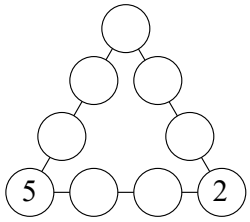


A 8 B 12 C 14 D 15 E 16

22. In the division calculation $952\,473 \div 18$, which two adjacent digits should be swapped in order to increase the result by 100?

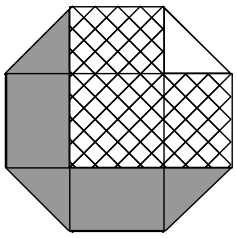
A 9 and 5 B 5 and 2 C 2 and 4 D 4 and 7 E 7 and 3

23. Sam wants to complete the diagram so that each of the nine circles contains one of the digits from 1 to 9 inclusive and each contains a different digit. Also, the digits in each of the three lines of four circles must have the same total. What is this total?



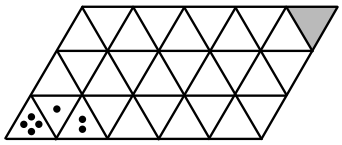
A 17 B 18 C 19 D 20 E 21

24. The diagram shows a regular octagon with sides of length 1. The octagon is divided into regions by four diagonals.
What is the difference between the area of the hatched region and the area of the region shaded grey?



A 0 B $\frac{1}{8}$ C $\frac{1}{4}$ D $\frac{1}{2}$ E 1

25. A die has the shape of a regular tetrahedron, with the four faces having 1, 2, 3 and 4 pips. The die is placed with 4 pips 'face down' in one corner of the triangular grid shown, so that the face with 4 pips precisely covers the triangle marked with 4 pips. The die is now 'rolled', by rotating about an edge without slipping, so that 1 pip is face down. It is rolled again, so that 2 pips are face down, as indicated. The rolling continues until the die rests on the shaded triangle in the opposite corner of the grid. How many pips are now face down?



A 1 B 2 C 3 D 4 E it depends on the route taken

UK JUNIOR MATHEMATICAL CHALLENGE

THURSDAY 29th APRIL 2010

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

http://www.ukmt.org.uk

SOLUTIONS LEAFLET

This solutions leaflet for the JMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

1. **B** The expression = 2010 + 2010 – 2010 – 2010 + 2010
= (2010 – 2010) + (2010 – 2010) + 2010 = 2010.

2. **E** In A, the letter T is incorrect; in B it is U which is incorrect; in C and D the incorrect letters are M and K respectively.

3. **A** 2010 mm = 2.01 m so, of the alternatives given, only a table could be expected to have a length of 2010 mm.

4. **D** Let X be on the top face of the cube. If the base is placed on a horizontal surface, then A, B, C, E will all be on vertical faces of the cube and D will be on the base, opposite X.

5. **D** Each of the five outer circles is divided into six regions, giving 30 regions in total. In addition, there is one region in the centre of the diagram and one region between the circles and the sides of the square. So, in all, there are 32 regions.

6. **C** The values of the expressions are A 12; B 15; C 16; D 15 and E 12.

7. **A** As 2, 5 and 10 are all factors of the correct product, this product is a multiple of 100. So the last digit and the last-but-one digit are both zero.

8. **D** If the mean of y and z is x, then $y + z = 2x$. So the sum of the interior angles of the triangle is $(x + y + z)° = 3x°$. So $3x = 180$, that is $x = 60$.

9. **A** One year is, at most, 366 days, so one-third of a year is less than 125 days. No month is longer than 31 days, so 4 months is also less than 125 days, as is 17.5 weeks which equals 122.5 days. However 3002 hours equals 125 days 2 hours, so this is the longest of the five periods of time.

10. **E** Third prize is worth one-sixth of the total prize money, so Mrs Keat received half of that amount, that is one-twelfth of the total.

11. **C** Divide the whole figure into horizontal strips of height 1 unit: its area is $(3 + 6 + 8 + 8 + 8 + 6 + 3)$ units^2 = 42 units^2. Similarly, the unshaded area is $(1 + 4 + 6 + 4 + 1)$ units^2 = 16 units^2. So the shaded area is 26 units^2.
Alternative solution: notice that if the inner polygon is moved a little, the answer remains the same – because it is just the difference between the areas of the two polygons. So, although we are not told it, we may assume that the inner one is so positioned that the outer shaded area can be split neatly into 1 by 1 squares – and there are 26 of these.

12. **C** There are 36 people to be seated so at least five tables will be required. The number of circular tables must be even. However, five rectangular tables will seat 40 people and three rectangular and two circular will seat 34. So at least six tables are needed. Two rectangular and four circular tables do seat 36 people: so six is the minimum number of tables.

13. **B** It is necessary to find a route for which the line is broken the first time it passes through any intersection and solid when it passes through that intersection for the second time. Only the route which starts at B and heads away from D satisfies this condition.

14. **D** The average number of vehicles per day $\approx \dfrac{300\,000\,000}{44 \times 365} \approx \dfrac{300\,000\,000}{40 \times 400}$
$= \dfrac{300\,000\,000}{16\,000} \approx \dfrac{300\,000\,000}{15\,000} = 20\,000.$

15. **C** The two shaded regions measure 3 by 7 and 1 by 6, so the total area outside the overlap is 27 units^2.

16. **E** As 108 marks represented 18% of the final total, 6 marks represented 1% of the final total. So this total was 600.

17. **D** As triangle *PQR* is equilateral, $x + 2y = 3x - y = 5y - x$. Equating any two of these expressions gives $2x = 3y$.
The only pair of given values which does not satisfy this equation is $x = 10$, $y = 6$.

18. **D** The other times that this has happened previously are when Sam's age in years went from 1 to 2; from 4 to 5; from 16 to 17 and from 36 to 37.
Note that since primes other than 2 are odd, the only squares which need to be checked, other than 1, are of even numbers.

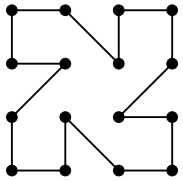
19. **C** Villages which have more than two roads leading to them (or from them) must all be visited more than once as a single visit will involve at most two roads. So Bentonville, Pencaster and Wytham must all be visited more than once. The route Home, Bentonville, Greendale, Wytham, Bentonville, Pencaster, Home, Wytham, Horndale, Pencaster, Home starts and finishes at Home and visits both Greendale and Horndale exactly once so the minimum number of villages is three.

20. **B** The seven numbers must total 49 if their mean is to be 7. The largest possible number will occur when the other six numbers are as small as possible, that is 1, 2, 3, 4, 5, 6. So the required number is $49 - 21 = 28$.

21. **C** The first and last hexagons both contribute 5 cm to the perimeter of the pattern. Every other hexagon in the pattern contributes 4 cm to the perimeter. The first and last thus contribute 10 cm, so we need another $2000 \div 4 = 500$ hexagons. Therefore the total number of hexagons required is 502.

22. **E** The prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19. It is not possible for 2 to be one of the six numbers Kiran wrote down, since that would give one of the pairs an odd sum, whereas both of the other pairs would add up to an even number. The sum of the remaining 7 primes is 75 which is a multiple of 3. The sum of the six primes making up the three pairs must also be a multiple of 3 since each pair has the same total. So the odd prime not used in the six pairs must be a multiple of 3 too. Therefore 3 is the odd prime not used. So each pair totals $72 \div 3$, that is 24, and the pairs are $5 + 19$, $7 + 17$, $11 + 13$.

23. **E** The number of sides of the polygon is equal to the number of corners it has. As no dot is at more than one corner, the maximum number of corners is 16. So the maximum possible number of sides is 16, provided that a 16-sided figure may be drawn. The figure on the right shows one of several ways in which this can be achieved.



24. **B** In the 21st Century, to obtain a sequence of two years or more then either a 2 or a 0 must be repeated in each year, or the sequence include years such as 2011, 2033, 2044 etc. So the only sequence after that mentioned in the question will be from 2020 to 2030, but this is too short.
In the 22nd Century, either a 2 or a 1 must be repeated. The first such sequence is 2110 to 2129 which does include 20 years, one of which is 2120.

25. **A** The three-digit number RRR is equal to 111 multiplied by the single digit R. So $PQPQ \times R = 639027 \div 111 = 5757$. Now $PQPQ$ equals the two-digit number PQ multiplied by 101. So $PQ \times R = 5757 \div 101 = 57$. The only ways in which 57 may be expressed as the product of a two-digit number and a single digit are 57×1 and 19×3. So $P = 5, Q = 7, R = 1$ or $P = 1, Q = 9, R = 3$. In both cases, $P + Q + R = 13$.

UK JUNIOR MATHEMATICAL CHALLENGE

FRIDAY 6th MAY 2011

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

http://www.ukmt.org.uk

The Actuarial Profession
making financial sense of the future

SOLUTIONS LEAFLET

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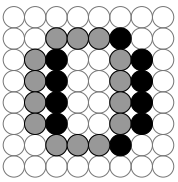
1. **B** $2 \times 0 \times 1 + 1 = 0 \times 1 + 1 = 0 + 1 = 1$.

2. **E** If the sum of the digits is a multiple of 3 then the number is a multiple of 3. The sums of the digits of the given numbers are 6, 9, 12, 15, 18, so they are all multiples of 3.
(*Can you prove that all numbers consisting of three consecutive digits are multiples of 3? Hint: let the second digit be n.*)

3. **B** In the diagram, the extra cells which need to be lit are shown in black.



So, in total, 24 cells are lit in a bold 'o'.

4. **C** 100 kg = 100 000 g. So the sum of money in £1 coins which would have the same mass as the world's largest coin is £(100 000 ÷ 10) = £10 000.
(*The coin was sold for* $4m (£2.6m) *at an auction in Vienna in June 2010.*)

5. **B** One third is equal to four twelfths. Hence the children ate eight twelfths of the bar between them. Each child ate one twelfth of the bar, so old Mother Hubbard had eight children.

6. **E** The six marked angles are the interior angles of the two large triangles which make up the star shape in the diagram, so their sum is $2 \times 180° = 360°$.

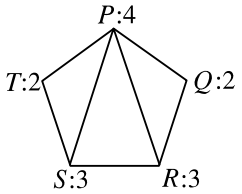
7. **D** There are 9 bushels in a barrel. Each bushel is 4 pecks, so there are 36 pecks in a barrel. Therefore 35 more pecks are needed.

8. **A** Let the original square have side $3x$. Then its perimeter is $12x$.
The perimeter of the octagon is $2 \times 4x + 3 \times 3x + 3 \times x = 8x + 9x + 3x = 20x$.

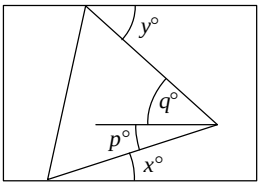
So the required ratio is $12:20 = 3:5$.

9. **A** $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$ is the sum of the digits of each such number. As 45 is a multiple of 9, each such number is a multiple of 9 and so too is the difference between two of them. Thus the smallest feasible difference is 9. The two numbers 123 456 798 and 123 456 789 show that this can occur.

10. **C** The diagram shows the number of lines which meet at the vertices P, Q, R, S, T. When the path around the diagram passes through a vertex, it uses up two of the edges. So, apart from the first and last vertex used, each vertex must have an even number of edges meeting at it. So we are obliged to use R or S as the first vertex, and the other as the last. The path $RQPTSRPS$, together with its reverse, shows that either is a possible start. (It is a fact that such a path can be drawn through a connected graph precisely when either all, or all but 2, vertices have an even number of edges meeting there.)



11. **C** A line segment which is parallel to two sides of the rectangle has been added to the diagram, as shown. The angle marked $p°$ is equal to the angle marked $x°$ as these are alternate angles between parallel lines. So $x = p$. Similarly $y = q$. The angles marked $p°$ and $q°$ together form one interior angle of an equilateral triangle. Therefore $x + y = p + q = 60$.

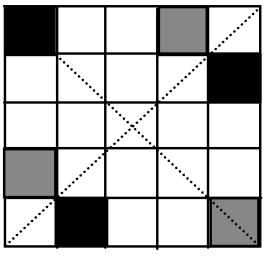


12. **E** ● = ■ + ▲ = ▲ + ▲ + ▲ = 3▲. Therefore ♦ = ● + ■ + ▲ = 3▲ + 2▲ + ▲ = 6▲.

13. **E** The mean of $\frac{2}{3}$ and $\frac{4}{9}$ is $\left(\frac{2}{3} + \frac{4}{9}\right) \div 2 = \left(\frac{6}{9} + \frac{4}{9}\right) \div 2 = \frac{10}{9} \div 2 = \frac{5}{9}$.
(*Note that the mean of two numbers lies midway between those two numbers.*)

14. **A** Let the area of the shaded face be x cm^2. Then the cuboid has two faces of area x cm^2 and four faces of area $4x$ cm^2. So its total surface area is $18x$ cm^2.
Therefore $18x = 72$, that is $x = 4$.
So the area of one of the visible unshaded faces is 4×4 cm^2 $= 16$ cm^2.

15. **A** In order that the figure has rotational symmetry of order 2, the three squares which appear in black must be shaded. When this has been done, we note that the broken lines shown are both lines of symmetry. So the minimum number of squares which must be shaded is 3.



16. **B** The smallest possible number of votes the winner could receive corresponds to the situation in which the numbers of votes received by each of the candidates are as close together as possible.
As $83 \div 4 = 20.75$, at least one of the candidates receives 21 votes or more. However, it is not possible for the winner to receive 21 votes, since there are still 62 votes to be allocated which makes it impossible for each of the other three candidates to receive fewer than 21 votes. So the winner must receive more than 21 votes. If the numbers of votes received by the candidates are 22, 21, 20, 20 then there is a winner and, therefore, 22 is the smallest number of votes the winner could receive.

17. **D** The lengths in minutes of the fifth set and the whole match are 491 and 665 respectively.
So the required fraction is $\frac{491}{665} = \frac{491 \times 3}{665 \times 3} \approx \frac{1500}{2000} = \frac{3}{4}$.

18. **C** Until Peri reaches Granny's, he travels 9m in every 10 days. So he takes 90 days to travel the first 81m of his journey. There remains a distance of 9m to be covered and so, after a further 9 days, Peri is at Granny's. Therefore the length of Peri's journey is 99 days, that is 14 weeks 1 day. So Peri arrives at Granny's on Tuesday.

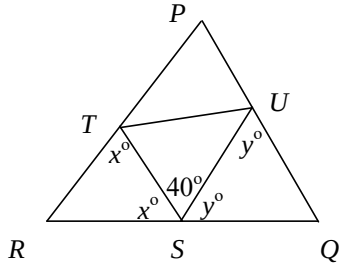
19. **D** Of the given numbers, 2, 3 and 5 are all prime and therefore appear in the list. In addition, 1 appears in the list as it is the units digit of 11 and also of many other primes. However, all numbers with units digit 4 are even and therefore not prime, because the only even prime is 2. So only 1, 2, 3, 5 appear in the list.

20. **A** Let the length of the side of each cube be x cm. Then the volume of the solid is $7x^3$ cm^3. Therefore $7x^3 = 875$, that is $x^3 = 125$. So $x = 5$. The surface area of the solid comprises five of the faces of each of six cubes. Each face has area 25 cm^2 so the required area is $5 \times 6 \times 25$ cm^2 $= 750$ cm^2.

21. **B** In total the train travels 27 km + 29 km = 56 km.
So the combined time for these two parts of the journey is $\frac{56}{96}$ hours = $\frac{7}{12}$ hours = 35 minutes.
The total journey time, therefore, is 38 minutes. So Gill arrives at 09:38.

22. **D** Let the numbers of stamps bought by Evariste and Sophie be x and y respectively. Then $1.1x + 0.7y = 10$, that is $11x + 7y = 100$. As 100 has remainder 2 when divided by 7, we need a multiple of 11 which is two more than a multiple of 7. The multiples of 11 less than 100 are 11, 22, 33, 44, 55, 66, 77, 88, 99. Of these only 44 is two more than a multiple of 7. So the only positive integer solutions of the Diophantine equation $11x + 7y = 100$ are $x = 4, y = 8$. Therefore Evariste buys 4 stamps, costing £4.40, and Sophie buys 8 stamps, costing £5.60.

23. **E** Let $\angle RTS = x°$. Then $\angle RST = x°$ as $RS = RT$.
Let $\angle QUS = y°$. Then $\angle QSU = y°$ as $QS = QU$.
As RSQ is a straight line, $x + y + 40 = 180$; so $x + y = 140$.



Now $\angle TPU = 180° - \angle TRS - \angle SQU$

$= 180° - (180 - 2x)° - (180 - 2y)°$

$= 180° - 180° + 2x° - 180° + 2y°$

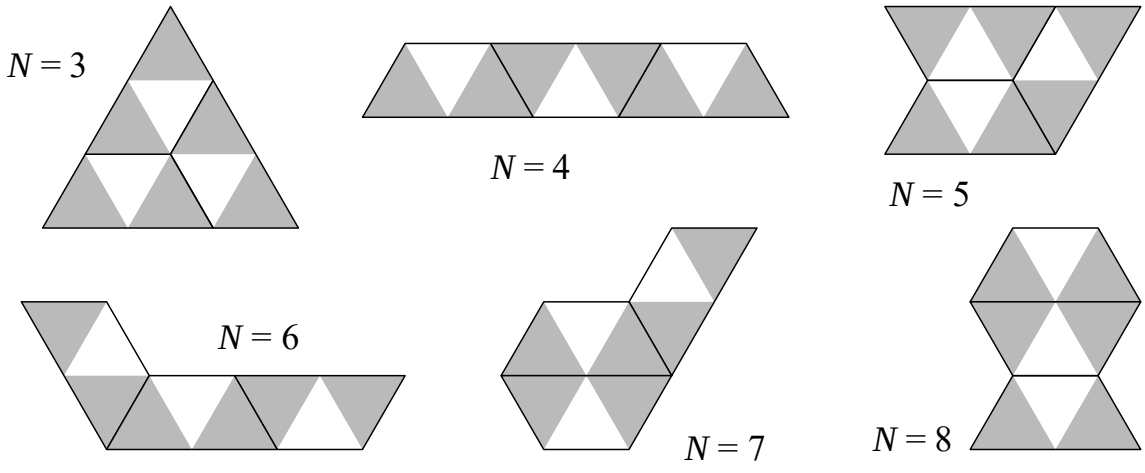
$= 2(x + y)° - 180°$

$= 2 \times 140° - 180°$

$= 100°$.

24. **D** (We may assume that the party is initially on the near bank and wishes to cross to the far bank.)
If an adult crosses to the far bank then there has to be a child waiting there to bring the raft back (unless an adult immediately brings the raft back − but this represents a wasted journey). This is possible only if the first two crossings involve both children crossing to the far bank and one of them staying there whilst the other brings the raft back. The third crossing involves the first adult crossing to the far bank and on the fourth crossing the child waiting on the far bank brings the raft back to the near bank. So after four crossings, one of the adults is on the far bank and the remainder of the party is on the near bank. This procedure is repeated so that after eight crossings, both adults are on the far bank and both children are on the near bank. A ninth and final crossing then takes both children to the far bank.

25. **C** The three trapezia have 12 edges in total. Whenever two trapezia are joined together the total number of edges is reduced by at least 2. Therefore the maximum possible value of N is $12 - 2 \times 2 = 8$. As the shapes form a polygon, N cannot be less than 3. The diagrams below show that all values of N from 3 to 8 are indeed possible, so there are 6 different values of N.



UK JUNIOR MATHEMATICAL CHALLENGE

THURSDAY 26th APRIL 2012

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

http://www.ukmt.org.uk

The Actuarial Profession
making financial sense of the future

SOLUTIONS LEAFLET

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1. **E** The smallest four-digit positive integer is 1000. Each of the subsequent integers up to and including 1022 has at least two digits the same. However, all digits of 1023 are different so this is the required integer.

2. **C** $1.01 \div 2 = 1 \div 2 + 0.01 \div 2 = 0.5 + 0.005 = 0.505$.

3. **E** An integer will have exactly one factor other than 1 and itself if, and only if, it is the square of a prime. Of the options given, the only such number is 25. Its factors are 1, 5, 25.

4. **C** None of the letters J, N, R has an axis of symmetry, so these letters cannot look the same when reflected in a mirror, no matter how the mirror is held. However, the letters U, I, O all have at least one axis of symmetry, so each may look the same when reflected in a mirror.

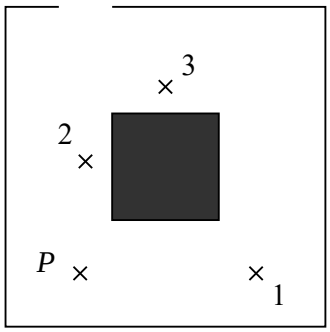
5. **D** $2012 - 1850 = 162$.

6. **D** The first two views of the cube show that I, M, U, O are not opposite K. So P is opposite K. Similarly, the second and third views show that I is opposite O. So the remaining two faces, M and U, must be opposite each other.

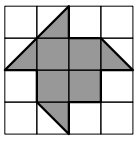
7. **B** Two medium cartridges can print as many pages as three small cartridges, i.e. 1800 pages. So three medium cartridges can print $1800 \times 3/2$ pages, i.e. 2700 pages. This is the same number of pages as two large cartridges can print, so one large cartridge can print $2700 \div 2$, i.e. 1350, pages.

8. **B** The 480 ml in Tommy's tankard represents three quarters of its capacity. So, one quarter of the capacity must be $480 \text{ ml} \div 3 = 160 \text{ ml}$.

9. **C** The person at position P can see exactly two of the other three people, so this person is Caz. The people he can see are at positions 1 and 2 and are Bea and Dan, each of whom can see exactly one person − Caz. This leaves Ali at position 3 − a position from which none of the three people can be seen, so all of the information given is consistent with Caz being at P.

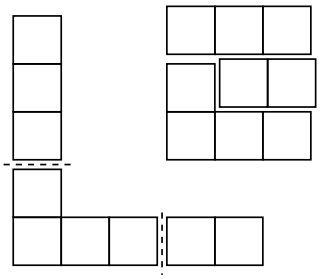


10. **E** The diagram shows the region of overlap, which has area 6 cm^2.



11. **B** We concentrate initially on the units digits of the numbers given, noting that the 3 comes first, so is positive. Now $3 + 7 = 10$ but there is no way to combine 5 and 9 to get a units digit 0. So we must use $3 - 7$. Hence, in the calculation, 67 must be preceded by a minus sign. Now $123 - 67 = 56$. So we need to get an extra 44 by combining 45 and 89. The only way to do this is $89 - 45$. So the correct calculation is $123 - 45 - 67 + 89$. It has two minus signs and one plus sign, so $p - m = 1 - 2 = -1$.

12. **B** None of the pieces which Laura uses to make the 3 × 3 square can be more than 3 units long. Both the horizontal and vertical portions of the original shape are longer than 3 units, so at least two cuts will be required. Hence Laura will need at least three pieces and the diagrams on the right show that the task is possible using exactly three pieces.



13. **C** Note that p is a factor of both 15 and 18. So p is either 1 or 3. If $p = 1$ then $w = 15$. However, if $w = 15$ then r is not an integer.
So $p = 3, w = 5, x = 6$. The values of the other input factors may now be calculated: $r = 8, s = 10, v = 2, z = 7, q = 5, y = 4, t = 6$.
So A + B + C + D + E = 6 + 25 + 48 + 40 + 42 = 161.

×	p	q	r	s	t
v	A	10		20	
w	15	B	40		
x	18		C	60	
y		20		D	24
z			56		E

14. **A** Note that 96 is a multiple of 6, so the 97th symbol is the same as the first, the 98th symbol is the same as the second and the 100th and 101st symbols are the same as the fourth and fifth symbols respectively.

15. **E** In total, the fraction of tulips which are either yellow or red is $\frac{1}{2} + \frac{1}{3} \times \frac{1}{2} = \frac{2}{3}$. So one third of the tulips are pink or white. Of these, one quarter are pink, so the fraction of tulips which are white is $\frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$.

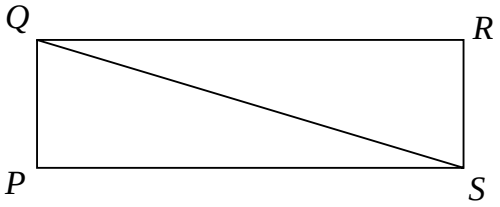
16. **D** Normally, Beth reads pages 1, 4, 7, ... ; Carolyn reads pages 2, 5, 8, ... ; George reads pages 3, 6, 9, ... When Beth is away, Carolyn reads all the odd-numbered pages, whilst George reads all the even-numbered pages. So the pages which are read by the person who normally reads that page are numbered 5, 11, 17 (Carolyn) and 6, 12, 18 (George).

17. **A** The number of boys in the class is $(24 - 6) \div 2 = 9$. So there are 9 boys and 15 girls.
Hence the required ratio is 5 : 3.

18. **D** Note that the number which replaces x appears in both the row and the column. Adding the numbers in the row and the column gives $2 + 3 + 4 + 5 + 6 + 7 + 8 + x = 2 \times 21 = 42$.
So $35 + x = 42$ and hence $x = 7$. The diagram shows one way in which the task may be accomplished.

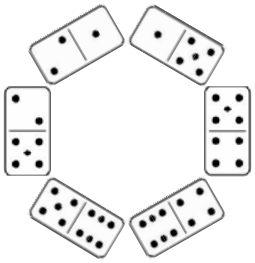
	2		
	4		
3	7	5	6
	8		

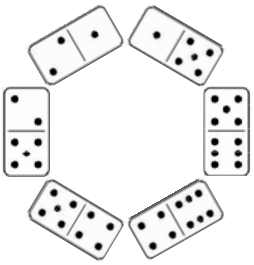
19. **E** As $\angle QPS = 90°$, $\angle PSQ + \angle PQS = 90°$.
So, since the ratio of these angles is 1:5,
$\angle PSQ = 15°$ and $\angle PQS = 75°$.
Now $\angle QSR = \angle PQS$ (alternate angles).
So $\angle QSR = 75°$.



20. **A** 50 months = 4 years and 2 months; 50 weeks and 50 days = 57 weeks and 1 day, i.e. just over 1 year and 1 month. So Aroon is just over 55 years and 3 months old and will, therefore, be 56 on his next birthday.

21. **B** Exactly two dominoes have a '1' and exactly two dominoes have a '2' so the dominoes  must be arranged as shown. So  cannot be adjacent to  . Clearly,  cannot be adjacent to  either, but it is possible to form a ring with  adjacent to  or with  adjacent to  . These are shown below. So only two of the dominoes cannot be placed adjacent to  .





22. **B** The original hexagon has been divided into seven regular hexagons and twelve equilateral triangles. Six equilateral triangles are equal in area to one smaller hexagon, so the large hexagon is equal in area to nine of the smaller hexagons. (This may also be deduced from the fact that their sides are in the ratio 3:1.) The shaded area consists of one smaller hexagon and six equilateral triangles, which is equivalent to the area of two of the smaller hexagons. So $\frac{2}{9}$ of the large hexagon is shaded.

23. **A** If either of the first two digits of the number is changed, the units digit will still be 0. Therefore the new number will be either 000 or a non-zero multiple of 10 and so will not be prime. If the units digit is changed then the possible outcomes are 201, 202, 203, 204, 205, 206, 207, 208, 209. The even numbers are not prime and neither are 201 (3 × 67), 203 (7 × 29), 205 (5 × 41), 207 (3 × 69), 209 (11 × 19).
So none of the numbers on Peter's list is prime.

24. **D** After 500 games, I have won $500 \times \frac{49}{100} = 245$ games. So I have lost 255 games. Therefore I need to win the next 10 games to have a 50% success rate.

25. **A** The sum of the interior angles of a triangle is 180°.
Therefore $5x + 3y + 3x + 20 + 10y + 30 = 180$, i.e. $8x + 13y = 130$.
As x and y are both positive integers, it may be deduced that x is a multiple of 13.
Also, since $y \geqslant 1$, $x \leqslant \frac{117}{8}$ so the only possible value of x is 13. If $x = 13$ then $y = 2$, so $x + y = 15$.

UK JUNIOR MATHEMATICAL CHALLENGE

THURSDAY 25th APRIL 2013

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

http://www.ukmt.org.uk

Institute and Faculty of Actuaries

SOLUTIONS LEAFLET

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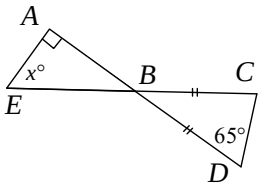
1. **E** All of the alternatives involve subtracting a number from 1. The largest result, therefore, will correspond to the smallest number to be subtracted, i.e. 0.00001.

2. **E** Their average height is $\frac{2.1 + 1.4}{2}$ m $= 1.75$ m.

3. **C** Triangle BCD is isosceles, so $\angle BCD = \angle BDC = 65°$.
The sum of the interior angles of a triangle is 180° so $\angle CBD = (180 - 2 \times 65)° = 50°$.
Therefore $\angle ABE = 50°$ (vertically opposite angles). So $\angle AEB = (180 - 90 - 50)° = 40°$.



4. **C** Distance travelled = average speed × time of travel, so Gill travelled between 15 km and 20 km. Of the alternatives given, only 19 km lies in this interval.

5. **D** The diagram shows the four lines of symmetry.

6. **A** $((1 - 1) - 1) - (1 - (1 - 1)) = (0 - 1) - (1 - 0) = -1 - 1 = -2$.

7. **D** Let the number of balls collected by Roger be x. Then Andy collects $2x$ balls and Maria collects $(2x - 5)$ balls. So $x + 2x + 2x - 5 = 35$, i.e. $5x = 40$, i.e. $x = 8$. So Andy collected 16 balls.

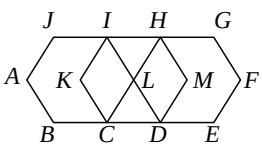
8. **A** The number 3 on the top ruler (which is 7cm from the left-hand end) aligns with the 4 on the bottom one (which is 6cm from the right-hand end). Thus $L = 7 + 6 = 13$.

9. **B** Let there be b boys and g girls in the family. Then Peter has g sisters and $(b - 1)$ brothers. So $g = 3(b - 1)$. Louise has $(g - 1)$ sisters and b brothers. So $g - 1 = 2b$. Therefore $2b + 1 = 3b - 3$, i.e. $b = 4$. So $g = 9$.
Therefore there are 4 boys and 9 girls in the family, i.e. 13 children in total.

10. **E** The top and bottom faces of the stack and the two touching faces form two pairs of opposite faces.
So the total number of pips on these four faces is $2 \times 7 = 14$. Therefore the total number of pips on the top and bottom faces of the stack is $14 - 5 = 9$.

11. **C** After Usain has run 100 m, his mum has run 50 m and Turbo has 'run' 10 m. So the distance between Usain's mum and Turbo is 40 m.

12. **E** Figure $ABEFGJ$ itself is a hexagon. There are three hexagons congruent to $ABCLIJ$; two hexagons congruent to $ABDMHJ$; four hexagons congruent to $ABCKIJ$; two hexagons congruent to $ABDLHJ$. So in total there are twelve hexagons.



13. **D** After the first coat, half of the paint is left. So after the second coat, the volume of paint remaining is one third of half of the capacity of the tin, i.e. one sixth of three litres = 500 ml.

14. **B** Let the two equal sides of the isosceles triangle have length a and the other side have length b. Then $2a + b = 20$. Since the sum of the lengths of any two sides of a triangle is greater than the length of the third, $2a > b$. Hence $4a > 2a + b$. So $4a > 20$, i.e. $a > 5$. Also $a < 10$ since $2a + b = 20$. So the possibilities are $a = 6$, $b = 8$; $a = 7, b = 6$; $a = 8, b = 4$; and $a = 9, b = 2$.

15. **A** When he starts to come down the hill, the Grand Old Duke of York has 90% of his men left. He loses 15% of these, so at the bottom of the hill he has 85% of 90% of the original number left. As $\frac{85}{100} \times 90 = 76\frac{1}{2}$, this means that $76\frac{1}{2}$% of his men were still there when they reached the bottom of the hill.

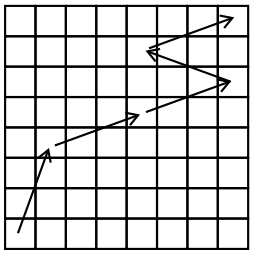
16. **B** The sum of the ages of the four children is $12 + 14 + 15 + 15 = 56$. Each year on their birthday, this sum increases by 4. So the number of years before the sum reaches 100 is $(100 - 56) \div 4 = 11$. Therefore their ages will first total 100 in 2024.

17. **E** Let x cm be the length of the $\smile$ shape. Although x is not given, it is clear that $x > 1$. The lengths, in cm, of the perimeters of pieces A, B, C, D, E are $4 + 6x$, $2 + 10x$, $7 + 5x$, $6 + 6x$, $1 + 11x$ respectively. As $4 + 6x < 6 + 6x$, the piece with the longest perimeter is B, C, D or E. As $x > 1$, it may be deduced that $7 + 5x < 6 + 6x < 2 + 10x < 1 + 11x$, so E has the longest perimeter.

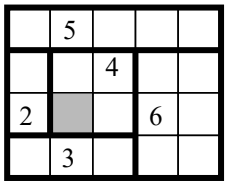
18. **A** Let the weights, in kg, of baby, nurse and me be x, y, z respectively. Then $x + z = 78$; $x + y = 69$; $y + z = 137$. Adding all three equations gives $2x + 2y + 2z = 284$, so $x + y + z = 284 \div 2 = 142$.
(*To find the combined weight, it is not necessary to find the individual weights, but baby weighs 5kg, nurse weighs 64 kg and I weigh 73 kg.*)

19. **D** For every 2 senior members in the swimming club there are 3 junior members. For every 5 senior members there are 2 veteran members. The lowest common multiple of 2 and 5 is 10, so it may be deduced that the number of senior members is a multiple of 10. For every 10 senior members in the swimming club there are 15 junior members and 4 veteran members. So the total number of members is a multiple of 29. Of the alternatives given, the only multiple of 29 is 58.

20. **B** The 'long knight' needs to move exactly seven squares to the right and exactly seven squares upwards. Although it is possible to move seven squares to the right in three moves (1, 3 and 3), in doing so it could move upwards by a maximum of five squares (3, 1 and 1). Similarly, it could move seven squares upwards in three moves, but could then move a maximum of five squares to the right. In four moves, the number of squares moved to the right must be even, since it is the sum of four odd numbers. So at least five moves are required and the diagram shows one way in which the task may be achieved in five moves.



21. **C** As 5 is a prime number, it must lie in a 5 × 1 rectangle. So the only possibility is the rectangle which covers the top row of the grid. Now consider 6: there is insufficient room for a 6 × 1 rectangle so it must lie in a 3 × 2 rectangle. There are only two such rectangles which include 6 but do not include either 4 or 3. If 6 comes in the middle of the top row of a 2 × 3 rectangle then there is space for a 3 × 1 rectangle including 3. But then there is not enough space for a rectangle including 4. So 6 must be placed in the rectangle shown. There is now insufficient room to place 4 in a 4 × 1 rectangle so it must lie in the 2 × 2 square shown, which includes the shaded square. This leaves the grid to be completed as shown.



22. **E** The diagram shows the totals of the rows and columns. The circled numbers are the total of the numbers in the two main diagonals. Note, by considering the average values of the rows and columns, that each should total 34. Row 2 and column 2 are both 2 short. So their common entry, 13, needs to increase by 2. So 13 must be interchanged with 15. (This change also reduces row 4 and column 3 by 2 and increases the main diagonal by 2, thus making all the sums equal 34 as desired.) So the sum of the numbers to be swapped is 28.

9	6	3	16	34
4	13	10	5	32
14	1	8	11	34
7	12	15	2	36
(34)	34	32	36	34 (32)

23. **D** Let the points awarded for a win and a draw be w and d respectively. Then $7w + 3d = 44$. The only positive integer solutions of this equation are $w = 2$, $d = 10$ and $w = 5$, $d = 3$. However, more points are awarded for a win than for a draw so we deduce that 5 points are awarded for a win and 3 points for a draw. So the number of points gained by my sister's team is $5 \times 5 + 2 \times 3 = 31$.

24. **B** Each of the overlapping areas contributes to the area of exactly two squares. So the total area of the three squares is equal to the area of the non-overlapping parts of the squares plus twice the total of the three overlapping areas i.e. $(117 + 2(2 + 5 + 8))\ \text{cm}^2 = (117 + 30)\ \text{cm}^2 = 147\ \text{cm}^2$.
So the area of each square is $(147 \div 3)\ \text{cm}^2 = 49\ \text{cm}^2$. Therefore the length of the side of each square is 7 cm.

25. **C** By arranging the tiles in suitable positions it is possible to place the 1 × 1 spotted square in any one of four corners of the steel sheet and then to place the grey square in any one of the other three corners. The other two corners will then be occupied by black squares. So, in total, there are 4 × 3 = 12 different looking installations.)

UK JUNIOR MATHEMATICAL CHALLENGE

THURSDAY 1st MAY 2014

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

http://www.ukmt.org.uk

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SOLUTIONS LEAFLET

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The UKMT is a registered charity

1. **D** $(999 - 99 + 9) \div 9 = (900 + 9) \div 9 = 909 \div 9 = 101$.

2. **B** There are 24 hours in one day, so $\frac{1}{12}$ of a day is 2 hours. Therefore the number of minutes in $\frac{1}{12}$ of a day is $2 \times 60 = 120$.

3. **C** The seats between us are T18 to T38 *inclusive*, that is, all the seats before seat 39 except for seats 1 to 17. So the number of seats is $38 - 17 = 21$.

4. **E** $987\,654\,321 \times 9 = 8\,888\,888\,889$.

5. **A** The smallest 4-digit number is 1000 and the largest 3-digit number is 999. They differ by 1.

6. **A** Let the width of each strip be 1. Then the square has side 5 and perimeter 20. The grey strips contribute 4 to the perimeter, so the fraction of the perimeter which is grey is $\frac{4}{20} = \frac{1}{5}$.

7. **B** $2002 - 4102 = -2100$. So $2014 - 4102 = -2100 + 12 = -2088$.

8. **A** Prime numbers have exactly two distinct factors, so 1 is not a prime number as it has exactly one factor. Of the others, 12, 1234 and 123 456 are all even numbers, so are not prime as the only even prime is 2. Also, $123 = 3 \times 41$ and 12 345 is clearly a multiple of 5, so neither of these is prime. Therefore none of the numbers in the list is prime.

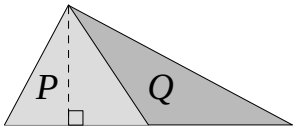
9. **E** The area of a triangle $= \frac{1}{2} \times$ base $\times$ height. If we let the length of the sides of each square in the grid be 1, then the area of triangle PQR is $\frac{1}{2} \times 3 \times 2 = 3$. The area of triangle XYZ is $\frac{1}{2} \times 6 \times 3 = 9$. So the required fraction is $\frac{3}{9} = \frac{1}{3}$.

10. **D** The angles at a point sum to 360°, so the largest angle in the triangle which includes the angle marked $x°$ is equal to $(360 - 90 - 90 - 60)° = 120°$. This triangle is isosceles as the sides of the three squares in the figure are equal to the sides of the equilateral triangle. So the triangle has angles 120°, $x°$ and $x°$. Therefore $x = \frac{1}{2}(180 - 120) = 30$.

11. **D** The third term of the sequence equals $1 + 2 = 3$. Now consider the fourth term: it is the sum of the first three terms. However, as the first two terms sum to the third term, the sum of the first three terms is twice the third term, i.e. $2 \times 3 = 6$. So the fourth term is twice the third term. Similar reasoning applies to each subsequent term, i.e. each term after the third term is equal to twice the term which precedes it. Therefore the sequence is 1, 2, 3, 6, 12, 24, 48, 96, ….

12. **B** As $7Q2ST - P3R96 = 22222$, it follows that $7Q2ST = P3R96 + 22222$. Looking at the units column: $2 + 6 = T$, so $T = 8$. Looking at the tens column, as $2 + 9 = 11$, we deduce that $S = 1$ and that 1 is carried to the hundreds column. Looking at the hundreds column: the carry of $1 + 2 + R$ must equal 12 since the sum has 2 in the hundreds column. So $R = 9$ and there is a carry of 1 to the thousands column. Looking at this column: the carry of $1 + 2 + 3 = Q$, so $Q = 6$. Finally, since there is no carry to the next column, $2 + P = 7$, so $P = 5$. Therefore the calculation is $76218 - 53996 = 22222$ and $P + Q + R + S + T = 5 + 6 + 9 + 1 + 8 = 29$.

13. **A** The diagram shows part of the given diagram after a rotation so that the diagonal shown is horizontal. The perpendicular height of triangle P is shown and it can be seen that this is also the perpendicular height of triangle Q. The diagonals of a rectangle bisect each other, so triangles P and Q have bases of equal length and the same perpendicular height. Therefore their areas are equal.



14. **D** One million millimetres is (1 000 000 ÷ 1000) m = 1000 m = 1 km.

15. **E** Consider, for example, the bottom left-hand corner of the envelope (see Figure 1). The two flaps overlap, so that the sum of the angles marked x and y is greater than 90°.
So when the flaps are unfolded, as in Figure 2, the angle marked z is less than 180°.
Therefore the correct answer is E.

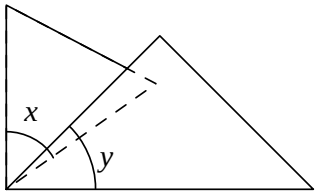


Figure 1

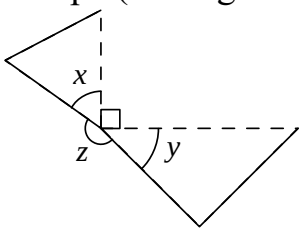
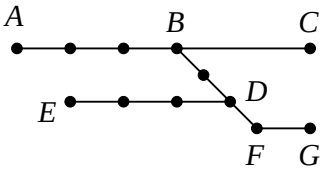


Figure 2

16. **E** If A is true then B is true which cannot be so since we are told only one statement is true. Hence A is false which is what E says. So E is the one true statement. [For completeness, we note that C and D must be false because we are told that exactly one statement is true; and B is false because A is false.]

17. **C** Whichever route is chosen, it must include section BD. We will divide the route into two sections. The first will include stations A, B, C and will finish at D. The second will start at D and include stations E, F, and G.



Clearly the first section cannot be traversed without visiting at least one station more than once and the route $A – B – C – B – D$ visits only B more than once so it is an optimal solution. Also, traversing the second section involves visiting D more than once as two branches lead from it. If $D – E – D$ is part of the route then two stations are visited more than once. However, if $D – G – D$ is part of the route then only F is visited more than once. So to traverse the second section, it is necessary to visit at least two stations (one of which is D) more than once. Therefore, the complete route must involve visiting at least 3 stations more than once. An example of an optimum route is $A – B – C – B – D – F – G – F – D – E$. The stations visited twice are B, D, and F.

18. **E** The units digit of any power of 5 is 5 so the units digit of $1 + 5^6$ is 6. Therefore the units digits of the calculations in the 5 options are 2, 1, 0, 9, 8 in that order. So the only calculation which could be correct is E. Checking this gives $1 + 5^6 - 8 = 1 + 15\,625 - 8 = 15\,626 - 8 = 15\,618$.

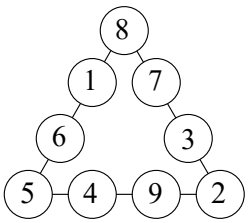
19. **C** Since Jack won 4 games, Jill lost 4 games for which she was awarded 4 points. So the number of games she won is (10 − 4) ÷ 2 = 3. Therefore, they played 7 games in total.

20. **B** Let the smallest number of chocolates required be n. Then $q + n > p - n$, that is $2n > p - q$. Therefore $n > \frac{1}{2}(p - q)$. Since $p > q$ and p and q are both odd, $\frac{1}{2}(p - q)$ is a positive integer. So the smallest possible value of n is $\frac{1}{2}(p - q) + 1 = \frac{1}{2}(p - q + 2)$.

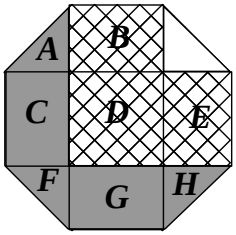
21. **D** Both the top and bottom layers of 9 cubes can be seen to contain 5 cubes with at least one face printed grey. The bottom layer could contain more than 5. In the middle layer, two cubes with grey faces are visible and there could be more. Therefore at least 12 cubes must have at least one face painted grey, which means that the largest number of cubes which Pablo can leave with no faces painted grey is $27 - 12 = 15$.

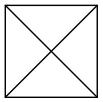
22. **C** In order to increase the result of the calculation (the quotient) by 100, the number to be divided (the dividend) must be increased by 100×18, that is 1800. So the new dividend needs to be $952\,473 + 1800$, that is 954 273. So the two digits which need to be swapped are 2 and 4.

23. **D** Note first that the sum of the first 9 positive integers is 45. Therefore, when the four numbers in each of the three lines are added together the total is 45 plus the sum of the numbers in the three corner circles, each of which contributes to the sum of two lines of circles. So if the number in the top circle is x, the total of all 3 lines is $45 + 2 + 5 + x = 52 + x$. As all three lines of circles must have the same total, $52 + x$ must be a multiple of 3. The possible values of x are 2, 5 and 8 but 2 and 5 have already been assigned. So $x = 8$ and the sum of each line is $60 \div 3 = 20$. The diagram shows one way of completing the task.



24. **C** Note that rectangles B, C, E and G are all congruent. Two of these are shaded grey and two are hatched, so the difference between the area of the hatched region and the area shaded grey is the difference between the area of square D of side 1 and the sum of the areas of triangles A, F and H. These are all isosceles right-angled triangles with hypotenuse 1 and the lower diagram shows how a square of side 1 may be divided into 4 such triangles. So the required difference in area is $1 - \frac{3}{4} = \frac{1}{4}$.





25. **A** If the die is rolled around a single vertex it covers, in turn, 6 small triangles making up a regular hexagon. It uses three different faces, repeated twice. An example is shown on the right. However, if it is rolled out from that hexagon in any direction, that will use the fourth face. The face that ends up covering each small triangle in the grid is always the same, regardless of the path taken to reach that triangle. Using these facts, it is easy to complete the diagram as shown. So, whichever route through the grid is taken, the '1' is face down when it reaches the shaded triangle.