

CPA Approach (Concrete, pictorial, Abstract)

### **Doubling and repeated addition**

Understand multiplication is related to doubling and combing groups of the same size (repeated addition) for 2, 5, 10.

Washing line, and other practical resources for counting. Concrete objects: Base Ten, Numicon, bundles of straws, bead strings

### **Bar Modelling**

Use of bar model as a visual to support early multiplication. This can be taught through cubes or counters

### **Problem solving**

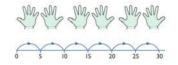
Problem solving with concrete objects (including money and measures) Use arrays to begin to understand multiplication can be done in any order (commutative)

### Vocabulary

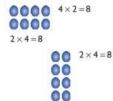
Groups of, times, multiply, addition, arrays, lots of



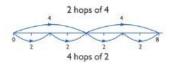
2 + 2 + 2 + 2 + 2 = 10 2 × 5 = 10 2 multiplied by 5 5 pairs 5 hops of 2



5+5+5+5+5+5=30  $5\times 6=30$ 5 multiplied by 6 6 groups of 5 6 hops of 5



 $4 \times 2 = 8$ 



| 4 | 4 | 4 |
|---|---|---|
|   |   |   |

3 X 4 = 12





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### **Times table Facts**

Recall and use multiplication facts for the 2, 5 and 10 multiplication tables as a minimum requirement Develop understanding of solving multiplication problems using arrays, objects, pictorial representations and number lines (see Year 1).

### **Number Sentences**

Expressing multiplication as a number sentence using x and explore commutative law of multiplication

### **Scaling**

Begin to develop understanding of multiplication as scaling (3 times bigger/taller)

### **Number lines**

Using number lines with a link to repeated addition.

### **Doubling numbers up to 10 + 10**

Link with understanding scaling Using known doubles to work out double 2digit numbers (double 15 = double 10 + double 5)

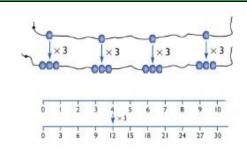
#### **Towards written methods**

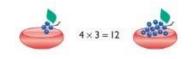
Use arrays and jottings to develop an understanding of doubling two digit numbers

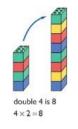
### Solve missing number problems.

### **Vocabulary**

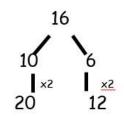
Groups of, times, multiply, addition, arrays, lots of, multiplication













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### **Mental methods**

Doubling 2 digit numbers using partitioning

Demonstrating multiplication on a number line – jumping in larger groups of amounts 13 x 4 = 10 groups of 4 then 3 groups of 4

Recall and use multiplication facts for the 3, 4 and 8 multiplication tables

### **Bar modelling**

Developing this method further to go alongside calculations

# Written methods (progressing to 3digit x 1digit)

Developing written methods using understanding of visual images to group and create equal groups of objects and pictures

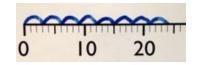
### **Grid method**

2 and 3 digit by 1 digit only looking at partitioning of multiplicand

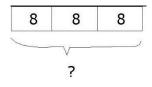
### **Short Multiplication**

2 by 1 digit numbers using known facts to support









$$3 \times 8 = ?$$

| × | 30 | 5 |
|---|----|---|
| 7 |    |   |

24 × 6 becomes

Answer: 144

### **Vocabulary**

Multiplicand (Number to be multiplied) Multiplier (Amount that multiplicand is being multiplied by) Product, lots of, groups of, times, as much, factors



CPA Approach
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### **Mental methods**

Counting in multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.

Recall and use multiplication facts for the 6, 7, 9, 11 and 12 multiplication tables
Use known facts to multiply by multiples of 10

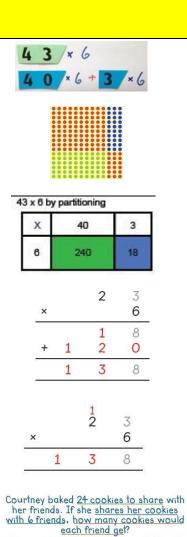
## Written methods (progressing to 3digit x 2digit, including 1dp)

Children to embed and deepen their understanding to multiply up to 2d x 2d progressing to 3d x 2d and decimals to 1dp. Ensure this is still linked back to their understanding of arrays and supported using place value counters

## <u>Leading to short written method</u> including regrouping (carrying):

Children to use their knowledge of multiplication tables and inverse, supported by pictorial representations and the Bar Method

## Bar Model to support calculations and word problems



C = amount of cookies for each friend

1022 × 4 =

1022 miles

Vocabulary

Multiplicand (Number to be multiplied) Multiplier (Amount that multiplicand is being multiplied by) Product, lots of, groups of, times, as much, factors



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### **Mental methods**

X whole numbers and decimals by 10, 100, 1000 using knowledge of place value to move digits

Use practical resources and jottings to explore equivalent statements (e.g.  $4 \times 35 = 2 \times 2 \times 35$ )

Recall of prime numbers up to 19 and identify prime numbers up to 100 (with reasoning)

Identify multiples and factor pairs for numbers

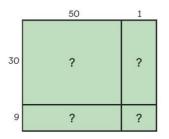
## Written methods (progressing to 4d x 2d)

Children to continue to explore long and short methods:

As in Year 4, children to use their knowledge of multiplication tables and inverse, supported by pictorial representations and the Bar Method

Grid method to multiply 2 digit numbers by 2 digit numbers e.g. 39 x 51 =

### Bar model to support Problem solving. Link with other operations



Jason rode with his uncle for 6 hours/at the average rate of 49 miles per hour./ How far did he travel?/

Jason travelled 294 miles.

### **Vocabulary**

Multiplicand (Number to be multiplied) Multiplier (Amount that multiplicand is being multiplied by) Product, lots of, groups of, times, as much, Factors



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#### **Mental methods**

jottings

Identifying common factors and multiples of given numbers and prime numbers X 2d and 3d numbers by 1d mentally or using

Perform mental calculations including mixed operations and large numbers

| Written | methods |
|---------|---------|
|---------|---------|

Continue to refine and deepen understanding of written methods including grid method, expanded column and fluency for using column multiplication supported by jottings and the Bar Method

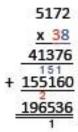
# Bar model to support word problems and percentages

| Х  | 1000  | 300  | 40  | 2  |
|----|-------|------|-----|----|
| 10 | 10000 | 3000 | 400 | 20 |
| 8  | 8000  | 2400 | 320 | 16 |

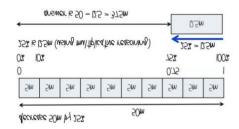
$$20 \times 113 = 2260$$

$$3 \times 113 = 339$$

$$23 \times 113 = 2599$$



by putting on both the perc and decimal number line you can link to multipliers, in this case it shows why the multiplier for a 25% decrease is 0.75



increase/decrease by a percentage

### Vocabulary

Multiplicand (Number to be multiplied) Multiplier (Amount that multiplicand is being multiplied by) Product, lots of, groups of, times, as much, Factors



### **Year Group Objectives for Multiplication**

### Year 1

Pupils should be taught to:

• Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

### Year 2

Pupils should be taught to:

- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (x), division (÷) and equals (=) signs
- How that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

### Year 3

Pupils should be taught to:

- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects



### Year 4

Pupils should be taught to:

- Recall multiplication and division facts for multiplication tables up to  $12 \times 12$
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- Recognise and use factor pairs and commutativity in mental calculations
- Multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

### Year 5

Pupils should be taught to:

- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- Know and use the vocabulary of prime numbers, prime factors and composite (nonprime) numbers
- Establish whether a number up to 100 is prime and recall prime numbers up to 19 ♣ multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- Multiply and divide numbers mentally drawing upon known facts
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
   Mathematics key stages 1 and 2 33 Statutory requirements
- Recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3)
- Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.



### Year 6

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- Perform mental calculations, including with mixed operations and large numbers
   Identify common factors, common multiples and prime numbers
- Use their knowledge of the order of operations to carry out calculations involving the four operations
- Solve addition and subtraction multi-step problems in contexts, deciding which
  operations and methods to use and why Mathematics key stages 1 and 2 40
  Statutory requirements
- Solve problems involving addition, subtraction, multiplication and division
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

### **Micro Steps**

The year group markings relate to Mental Arithmetic sessions. To be used also in planning where appropriate to learning

| 1. 2 digit ×1 digit without  carrying. 2 times table  - m: kimum in top  44).  multiplier  E.g. 34 × 2 | 23 × 2<br>41 × 2   | Year 3 Term 2 |
|--|--------------------|---------------|
| 2. 3 digit ×1 digit without  carrying.  2 times tables  ximum in top  multipli  44).                   | 123 × 2<br>231 × 2 | Year 3 Term 2 |



| E.g 324 × 2  |   |                  |                                 |
|--|---|------------------|---------------------------------|
| 3. 2 digit × 1 digit (2 times table only) carrying into tens colum 49                      | TU<br>17 × 2<br>18 × 2  | Year 3 Ter       | m 2                             |
| E.g 26 × 2  4. 2 digit × 1 digit ( 2 time; table only) carrying in to both columns         | TU<br>87 × 2<br>58 × 2  | Year 3 Ter       | m 4                             |
| 76 × 2 Assessment of stages 1-7  |   |                  |                                 |
| 5. 2 digit × 1 digit ( <mark>2-9 times table) carrying in both column</mark> .             | 67 × 3<br>89 × 3  | Year 3 Ter       | m 4                             |
| E.g. 73 × 5  |   |                  |                                 |
| 6. 3 digit x 1 digit carrying across any colum s   | 434 × 5<br>743 × 6  | Year 3<br>Term 4 | Year 3<br>Term 6                |
| 345 × 6 7. 3 digit x 1digit carrying 0 across any columns with in tens Column  E.g 407 x 8 | T<br>503 × 4<br>909 × 2                                       | Year 3<br>Term 6 | Year 4 Term 2 Recap on learning |
| 8. Repeat steps as above with decimal numbers  E,g. 3.5 x 6                                | 1.7 × 2 8.7×2 8.9×6<br>4.34 × 5 5.03 × 4<br>74.3 × 6 90.9 × 2 | Year 3<br>Term 6 | Year 4 Term 2 Year 3 Term 6     |



| 9. Numbers with any leng the of digits x 1 digit. (including money)  E.g. 456.2 x 5             | 456.7 x 3<br>£41.50 x 3              | Year 4<br>Term 2 | Year 4 Term 4 Recap on learning |
|---|--------------------------------------|------------------|---------------------------------|
| Assessment of stages 5 - 9  |                                      |                  |                                 |
| 10. 2 digit x 2 digit (maxim ım value of multiplier 11 – 19)                                    | 48 x 12<br>56 x 15                   | Year 4 Term 4    |                                 |
| 23  |                                      |                  |                                 |
| 11. 2 digit x 2 digit when o ne of the multipliers is a multiple of 10                          | 63 x 10<br>95 x 20                   | Year 4 Term 4    |                                 |
|   |                                      |                  |                                 |
| E.g. 45 x 70  |                                      |                  |                                 |
| 12. 2 digit x 2 digit when the answer to the tens x units in the multiplier is a multiple of 10 | T u<br>25 x 37<br>45 x 28            | Year 4 Term 4    |                                 |
| E.g. 30 x 63  | 25<br><u>37</u><br>175<br>150<br>600 |                  |                                 |
| 13. 2 digit x 2 digit - any values  | 48 x 56<br>19 x 27                   | Year 4<br>Term 6 | Year 5<br>Term 2<br>Greater     |
| E.g. 36 x 74  |                                      |                  | Depth                           |



| 14. 3 digits x 2 digits with 0's in any place value of either number  E.g. 304 x 56 | 407 x 27<br>270 x 47  | Year 4<br>Term 6 | Year 5 Term 2 Greater Depth          |
|---|-----------------------|------------------|--------------------------------------|
| 15. 3 digit x 2 digit with 0's in any place value of either number  E.g. 402 x 67   | 306 x 37<br>508 x 28  | Year 4<br>Term 6 | Year 5<br>Term 2<br>Greater<br>Depth |
| Assessment of stages 10 -15   |                       |                  |                                      |
| 16. 2 digit (1dp) x 2 digit,<br>when final answer ends in<br>a 0 e.g. 176.0 = 176   | 4.5 x 34<br>6.5 x 68  | Year 5 Term 4    |                                      |
| E.g. 3.5 x 64   |                       | 1                |                                      |
| 17. 2 digits (1dp) x 2 digits. Any 2 numbers.                                       | 5.6 x 27<br>3.7 x 64  | Year 5<br>Term 6 | Year 6<br>Term 2                     |
| E.g. 4.7 x 86   |                       |                  |                                      |
| 18. 2 digit (1dp) x 3 digit. Any<br>2 numbers                                       | 234 x 7.3<br>356x 6.4 | Year 5<br>Term 6 | Year 6<br>Term 2                     |
| E.g. 453 x 6.2  |                       |                  |                                      |

| 19. Any Money amount x 2<br>digit number | £37.29 x 73<br>£ 53.95 x 84 | Year 6 Term 2 |
|--|-----------------------------|---------------|
| E.g. £43.95 x 62                         |                             |               |
| 20. 3 digit x 3 digit number             | 754 x 603<br>807 x 326      | Year 6 Term 2 |
| E.g. 643 x 706                           |                             |               |
| Assessment of stages 16 – 20             |                             |               |

